ABE MATHEMATICS SCOPE AND SEQUENCE

**Level Two-Unit One** GLE 4 - 6

Teachers: Maura McCabe, Ryan Lamey and Stacy Decker

Unit Title: Numbers and Operations: Base Ten and The Number System

Amount of time for the unit: 15 hours

Purpose of Unit: In this unit we will build on place value with multi-digit whole numbers and decimals in real life situations, such as budget and event planning. We will explore practical applications such as estimation, rounding and problems in which remainders must be interpreted. Our ESOL learners may be more comfortable using algorithms which they learned in their own countries. This will be acknowledged and encouraged.

The foundation for operations with fractions will be built as we work with factors and multiples.

Unit Goals and Outcomes-Students will be able to:

* Generalize place value understanding for multi-digit whole numbers
* Compare multi-digit numbers using > and < signs.
* Expand multiplication and division to include three digit divisors and correctly interpret remainders.
* Read, write and compare decimals to the thousandths.
* Round multi-digit numbers to any place and appropriately place numbers on a number line
* Use place value understanding and properties to perform multi-digit arithmetic
* Solve multi-step word problems or tasks of all operations with multiple strategies such as drawing a picture, using a table and using simplified numbers.
* Find factor pairs for whole numbers less than 100 and determine whether a number is prime or composite.
* Find the greatest common factor and the least common multiple of numbers less than 100.

Priority Standards:

Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. (4.NBT.2)

Use place value understanding to round multi-digit whole numbers to any place. (4.NBT.3)

Fluently add and subtract, multiply, and divide multi-digit whole numbers using the standard algorithm. (4.NBT.4) (5.NBT.5) (6.NS.2)

Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left. (5.NBT.1)

Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10. (5.NBT.2)

Read, write, and compare decimals to thousandths. (5.NBT.3)

• Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000). (5.NBT.3a)

• Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. (5.NBT.3b)

Use place value understanding to round decimals to any place. (5.NBT.4)

Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. (5.NBT.6)

Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. (5.NBT.7) [Note from panel: Applications involving financial literacy should be used.]

Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite. (4.OA.4)

Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).* (6.NS.4)

Coherence:

Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.16 (3.OA.8)

Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.* (3.OA.9)

Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.* (4.NBT.1)

Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between

multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. (4.NBT.6)

16 This standard is limited to problems posed with whole numbers having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order. (Order of Operations)

Standards for Math Practice:

MP.1 Make sense of problems and persevere in solving them.

MP.2 Reason abstractly and quantitatively.

MP.3 Construct viable arguments and critique the reasoning of others.

MP.4 Model with mathematics.

MP.5 Use appropriate tools strategically.

MP.6 Attend to precision.

MP.7 Look for and make use of structure.

MP.8 Look for and express regularity in repeated reasoning.

Assessment of Priority Standards and Key Content:

|  |  |
| --- | --- |
| Generalize place value understanding for multi-digit whole numbers |  |
| Compare multi-digit numbers using > and < signs. |  |
| Expand multiplication and division to include three digit divisors and correctly interpret remainders. | **Page 3-Mini Assessment from**  **Achieve the Core link below** |
| Read, write and compare decimals to the thousandths. |  |
| Round multi-digit numbers to any place and appropriately place numbers on a number line |  |
| Use place value understanding and properties to perform multi-digit arithmetic |  |
| Solve multi-step word problems or tasks of all operations with multiple strategies such as drawing a picture, using a table and using simplified numbers. | Teacher assigned task on balancing a checkbook, reading a pay stub and budget planning. |
| Find factor pairs for whole numbers less than 100 and determine whether a number is prime or composite. |  |
| Find the greatest common factor and the least common multiple of numbers less than 100. |  |

Main Texts and Tools of Instruction:

|  |  |
| --- | --- |
| Virginia DOE has annotated lesson plans aligned to the standards  <http://www.doe.virginia.gov/testing/solsearch/sol/math/5/mess_5-1.pdf> |  |
| Achieve the Core provides lessons with formative assessments  <http://achievethecore.org/page/1032/multi-digit-multiplication> |  |
| The Math Assessment Project provides lesson plans with formative assessments  <http://www.map.mathshell.org/lessons.php?unit=6110&collection=8> |  |

<https://www.illustrativemathematics.org/>

<http://www.commoncoresheets.com/Values.php>

<https://www.superteacherworksheets.com/factors.html> (By subscription)

Pages 2 - 16 High *School Equivalency Test Preparation: Mathematics*, Paxen, 2014

ABE MATHEMATICS SCOPE AND SEQUENCE

**Level Two-Unit Two** GLE 4 - 6

Teachers: Maura McCabe, Ryan Lamey and Stacy Decker

Unit Title: Numbers and Operations: Fractions

Amount of time for the unit: 20 hours

Purpose of Unit: This unit will build on the understanding of fractions as numbers introduced in Level One. Continuing to use concrete and pictorial models we seek to expand students comfort with fractions and mixed numbers. We will cover equivalents, comparisons, and all operations with modeling in practical applications. In addition, we will introduce finding fraction and decimal equivalents.

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Unit Goals and Outcomes-Students will be able to:

* Express whole numbers as fractions
* Extend understanding of fraction equivalence and ordering.
* Represent equivalent fractions on a number line and as same sized models.
* Find equivalent fractions by multiplying or dividing by *n/n*.
* Compare fractions with unlike numerators and denominators.
* Estimate fractions as close to 0, ½ or 1.
* Add and subtract fractions and mixed numbers with like and unlike denominators.
* Use multiplication to find the fraction of a number; whole, mixed number or another fraction.
* Divide fractions and mixed numbers
* Determine the correct operation to solve fraction word problems or tasks.
* Convert fraction and decimal equivalents

Priority Standards:

Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.

*Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point*

*of a number line diagram.* (3.NF.3c)

Explain why *a* fraction *a*/*b* is equivalent to a fraction (*n* × *a*)/(*n* × *b*) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. (4.NF.1)

Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.(4.NF.2)

Understand a fraction *a*/*b* with *a* > 1 as a sum of fractions 1/*b*. (4.NF.3)

(NOTE: See page 62 for full text)

Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d* *= (ad + bc)/bd.)* (5.NF.1)

Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that* *3/7 < 1/2.* (5.NF.2)

Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. (4.NF.4) (NOTE: See page 62 for full text)

Interpret a fraction as division of the numerator by the denominator (*a*/*b* = *a* ÷ *b*). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?* (5.NF.3)

Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. (5.NF.4)

Interpret multiplication as scaling (resizing), (5.NF.5) (NOTE: See page 63 for full text)

Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem. (5.NF.6)

Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. (5.NF.7) (NOTE: See page 64 for full text)

Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is* *2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?* (6.NS.1)

Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100;* *describe a length as 0.62 meters; locate 0.62 on a number line diagram.* (4.NF.6)

Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model. (4.NF.7)

Coherence:

Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a/b* as the quantity formed by a parts of size 1*/b*. (3.NF.1)

Understand a fraction as a number on the number line; represent fractions on a number line diagram. (3.NF.2) (NOTE: See page 55 for full text)

Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. (3.NF.3) (NOTE: See page 55 for full text)

Standards for Math Practice:

MP.1 Make sense of problems and persevere in solving them.

MP.2 Reason abstractly and quantitatively.

MP.3 Construct viable arguments and critique the reasoning of others.

MP.4 Model with mathematics.

MP.5 Use appropriate tools strategically.

MP.6 Attend to precision.

MP.7 Look for and make use of structure.

MP.8 Look for and express regularity in repeated reasoning.

Assessment of Priority Standards and Key Content:

|  |  |
| --- | --- |
| Express whole numbers as fractions | Pages 10 & 22 *The Meaning of Fractions* |
| Extend understanding of fraction equivalence and ordering. |  |
| Represent equivalent fractions on a number line and as same sized models. |  |
| Find equivalent fractions by multiplying or dividing by *n/n*. |  |
| Compare fractions with unlike numerators and denominators. |  |
| Estimate fractions as close to 0, ½ or 1. |  |
| Add and subtract fractions and mixed numbers with like and unlike denominators. |  |
| Use multiplication to find the fraction of a number; whole, mixed number or another fraction. |  |
| Divide fractions and mixed numbers |  |
| Determine the correct operation to solve fraction word problems or tasks. |  |
| Convert fraction and decimal equivalents |  |

Main Texts and Tools of Instruction:

|  |  |
| --- | --- |
| Achieve the Core provides lessons with summative assessments  <http://achievethecore.org/page/1056/fraction-concepts>  and  <http://achievethecore.org/page/1034/multiplication-and-division-of-fractions> |  |
| The Math Assessment Project provides lesson plans with formative assessments  <http://www.map.mathshell.org/lessons.php> |  |
| Illustrative Math has many tasks for formative assessment. This link is for an addition of fractions game.  <https://www.illustrativemathematics.org/content-standards/5/NF/A/1/tasks/1563> |  |
| Virginia DOE has annotated lesson plans aligned to the standards  <http://www.doe.virginia.gov/testing/solsearch/sol/math/5/mess_5-1.pdf>  <http://www.doe.virginia.gov/testing/solsearch/sol/math/5/mess_5-2ab.pdf> |  |
| <http://www.doe.virginia.gov/testing/solsearch/sol/math/4/mess_4-3d.pdf> |  |

Fraction pieces or strips

<http://www.visualfractions.com/>

<https://www.illustrativemathematics.org/>

<http://www.math-aids.com/>

[http://www.commoncoresheets.com/Fractions.php](http://www.commoncoresheets.com/Fractions.php%20)

Pages72- 88 *Changing the Way We Teach Math*, Kate Nonesuch, Malaspina University-College, 2006

Contemporary’s NUMBER SENSE: *Fraction Addition and Subtraction*, Allan Suter, 1990

Contemporary’s NUMBER SENSE: *Fraction Multiplication and Division*, Allan Suter, 1990

*Key to Fractions: Adding and Subtracting*, Steve Rasmussen, Key Curriculum Press, 1980

*Key to Fractions: Multiplying and Dividing*, Steve Rasmussen, Key Curriculum Press, 1980

*Key to Fractions: Mixed Numbers*, Steve Rasmussen, Key Curriculum Press, 1980

ABE MATHEMATICS SCOPE AND SEQUENCE

**Level Two-Unit Three** GLE 4 - 6

Teachers: Maura McCabe, Ryan Lamey, and Stacy Decker

Unit Title: Ratio and Proportion

Amount of time for the unit: 8 hours

Purpose of Unit: The purpose of this unit is to introduce ratio and unit rate as a comparison of two quantities and explore many applications such as in shopping rates, miles per hour or gallon, scale of miles etc. In addition, we introduce problems and tasks utilizing proportions in Level 2 which will be expand on in our units on Percent, Geometry, Measurement and Statistics.

Unit Goals and Outcomes-Students will be able to:

* Understand the concept of ratio and unit rate.
* Compare ratios and unit rates.
* Use ratio and unit rate to solve word problems or tasks

Priority Standards:

Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”* (6.RP.1)

Understand the concept of a unit rate *a*/*b* associated with a ratio *a*:*b* with b ≠ 0, and use rate language in the context of a ratio relationship. *For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar.” “We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger.”* (6.RP.2)

Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. (6.RP.3)

• Solve unit rate problems including those involving unit pricing and constant speed. *For example,* *if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?* (6.RP.3b)

• Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities. (6.RP.3d)

**NOTE**: 6.RP.3a is a Level 3 standard.

6.RP.3c addresses percent and will be addressed in our percent unit.

Recognize and represent proportional relationships between quantities. (7.RP.2)

• Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. (7.RP.2a)

Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.* (7.RP.3) [Also see 7.G.1 and G.MG.2]

**NOTE:** 7.RP.2 and 7.RP.3 will be introduced at this level, but expanded on in Level 3

Coherence:

Explain why *a* fraction *a*/*b* is equivalent to a fraction (*n* × *a*)/(*n* × *b*) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. (4.NF.1)

Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. (4.NF.2)

Standards for Math Practice:

MP.1 Make sense of problems and persevere in solving them.

MP.2 Reason abstractly and quantitatively.

MP.3 Construct viable arguments and critique the reasoning of others.

MP.4 Model with mathematics.

MP.5 Use appropriate tools strategically.

MP.6 Attend to precision.

MP.7 Look for and make use of structure.

Assessment of Priority Standards and Key Content:

|  |  |
| --- | --- |
| Understand the concept of ratio and unit rate. |  |
| Compare ratios and unit rates. |  |
| Use ratio and unit rate to solve word problems or tasks |  |

Main Texts and Tools of Instruction:

|  |  |
| --- | --- |
| The Math Assessment Project provides lesson plans with formative assessments  <http://www.map.mathshell.org/lessons.php?unit=6200&collection=8>  <http://www.map.mathshell.org/lessons.php?unit=6230&collection=8> |  |
| *Massachusetts Community College Workforce*  *Development Transformation Agenda* has developed contextualized modules for three key **Industry Sectors**—healthcare, advanced manufacturing and information technology.  <http://mccwdta.etlo.org> |  |

Graph paper for scale

<https://www.illustrativemathematics.org/>

<http://www.math-aids.com/>

<http://www.commoncoresheets.com/Values.php>

Pages 27 – 31 *High School Equivalency Preparation: Mathematics,* Paxen, 2014

ABE MATHEMATICS SCOPE AND SEQUENCE

**Level Two-Unit Four** GLE 4 - 6

Teacher: Maura McCabe, Ryan Lamey, and Stacy Decker

Unit Title: Percent

Amount of time for the unit: 10 hours

Purpose of Unit: The purpose of this unit is to introduce percent as a part of 100, acknowledging that it is a ratio and has decimal and fraction equivalents. We will use proportional relationships to solve percent word problems and tasks.

Unit Goals and Outcomes-Students will be able to:

* Represent per cent as parts per 100.
* Use the percent ratio to solve word problems and tasks.
* Find fraction, decimal and percent equivalents.

Priority Standards:

Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. (6.RP.3)

• Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. (6.RP.3c)

Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.* (7.RP.3) [Also see 7.G.1 and G.MG.2]

**NOTE:** Ratio and percent problems will be introduced at this level, but multi-step problems will be explored in Level 3.

Coherence:

Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. (6.RP.3)

• Solve unit rate problems including those involving unit pricing and constant speed. *For example,*

*if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?* (6.RP.3b)

Standards for Math Practice:

MP.1 Make sense of problems and persevere in solving them.

MP.2 Reason abstractly and quantitatively.

MP.4 Model with mathematics.

MP.5 Use appropriate tools strategically.

MP.6 Attend to precision.

MP.7 Look for and make use of structure.

Assessment of Priority Standards and Key Content:

|  |  |
| --- | --- |
| Represent per cent as parts per 100. | Identify percent shaded on Pages 92 – 107 of Changing The Way We Teach Math |
| Use the percent ratio to solve word problems and tasks. |  |
| Find fraction, decimal and percent equivalents. | Teacher or text generated tasks or Activities from *Math Assessment Project* listed below |
| Summative assessment |  |

Main Texts and Tools of Instruction:

|  |  |
| --- | --- |
| The *Math Assessment Project* provides lesson plans with formative assessments  <http://www.map.mathshell.org/lessons.php?unit=6120&collection=8> |  |
| *Changing the Way We Teach Math* by Kate Nonesuch  has shaded grids to identify common percent, decimal and fraction equivalents. |  |
| *Massachusetts Community College Workforce*  *Development Transformation Agenda* has developed contextualized modules for three key **Industry Sectors**—healthcare, advanced manufacturing and information technology.  <http://mccwdta.etlo.org> |  |

<https://www.illustrativemathematics.org/>

<http://www.math-aids.com/>

<http://www.commoncoresheets.com/Values.php>

Pages 96 – 122, *Contemporary’s Number Power, Fractions, Decimals, and Percents*, Jerry Howett, 2000

Pages 42 – 46 *High School Equivalency Preparation: Mathematics,* Paxen, 2014

ABE MATHEMATICS SCOPE AND SEQUENCE

**Level Two-Unit Five** GLE 4 - 6

Teachers: Maura McCabe, Ryan Lamey, and Stacy Decker

Unit Title: Algebra: Operations, Expressions and Equations

Amount of time for the unit: 15 hours

Purpose of Unit: Our Algebra unit builds on the Operations and Algebraic Thinking standards introduced in Level One. In addition, it integrates operations with whole number, fractions and decimals from previous units in Level Two. This unit extends the students understanding of the order of operations, inverse operations and variables. Students will write expressions, and write and solve equations and inequalities. Although the CCRSAE classifies the coordinate plane as Geometry, we introduce it in this unit along with negative integers to indicate direction.

Unit Goals and Outcomes-Students will be able to:

* Recognize and generate number and shape patterns.
* Evaluate numerical expressions according to the order of operations with exponents and square roots.
* Write and evaluate numerical expressions with variables by substituting the given values.
* Solve one variable equations and inequalities.
* Write equations and inequalities to represent real world situations.
* Plot points on the coordinate plane

Priority Standards:

Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule “Add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.* (4.OA.5)

Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols. (5.OA.1)

Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation “add 8 and 7, then multiply by 2” as 2 × (8 + 7). Recognize that 3 × (2100 + 425) is three times as large as the 2100 + 425, without having to calculate the indicated sum or product.* (5.OA.2)

Write and evaluate numerical expressions involving whole-number exponents. (6.EE.1)

Write, read, and evaluate expressions in which letters stand for numbers. (6.EE.2)

* Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation “Subtract y from 5” as 5 – y.* (6.EE.2a)
* Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.* (6.EE.2b)
* Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas V = s3 and A = 6 s2 to find the volume and surface area of a cube with sides of length s = 1/2.* (6.EE.2c)

Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties* *of operations to y + y + y to produce the equivalent expression 3y.* (6.EE.3)

Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are* *equivalent because they name the same number regardless of which number y stands for.* (6.EE.4 )

Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true. (6.EE.5)

Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. (6.EE.6)

Solve real-world and mathematical problems by writing and solving equations of the form *x* + *p* = *q* and *px* = *q* for cases in which *p*, *q* and *x* are all nonnegative rational numbers. (6.EE.7)

Write an inequality of the form *x* > *c* or *x* < *c* to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form *x* > *c* or *x* < *c* have infinitely many solutions; represent solutions of such inequalities on number line diagrams. (6.EE.8)

Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate). (5.G.1)

Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation. (5.G.2)

Coherence:

Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations. (4.OA.1)

Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. (4.OA.2)

Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (4.OA.3)

Standards for Math Practice:

MP.1 Make sense of problems and persevere in solving them.

MP.2 Reason abstractly and quantitatively.

MP.3 Construct viable arguments and critique the reasoning of others.

MP.4 Model with mathematics.

MP.5 Use appropriate tools strategically.

MP.6 Attend to precision.

MP.7 Look for and make use of structure.

MP.8 Look for and express regularity in repeated reasoning.

Assessment of Priority Standards and Key Content:

|  |  |
| --- | --- |
| Recognize and generate number and shape patterns. |  |
| Evaluate numerical expressions according to the order of operations with exponents and square roots. |  |
| Write and evaluate numerical expressions with variables by substituting the given values. |  |
| Solve one variable equations and inequalities.  Solve inequalities. |  |
| Write equations and inequalities to represent real world situations. |  |
| Plot points on the coordinate plane |  |

Main Texts and Tools of Instruction:

<https://www.illustrativemathematics.org/>

<http://www.math-aids.com/>

<http://www.commoncoresheets.com/Values.php>

[https://www.superteacherworksheets.com/full -math.html](https://www.superteacherworksheets.com/full%20-math.html) (by subscription)

Pages 172 -199, Contemporary’s NUMBER POWER REVIEW, Whole Numbers to Algebra, Robert Mitchell, 1993

ABE MATHEMATICS SCOPE AND SEQUENCE

**Level Two-Unit Six** GLE 4 - 6

Teachers: Maura McCabe, Ryan Lamey, and Stacy Decker

Unit Title: Geometry and Measurement

Amount of time for the unit: 15 hours

Purpose of Unit: The purpose of this unit is to expand on the introductions to Geometry and Measurement in Level One. Students will be able to describe properties and attributes of two and three-dimensional figures. They will measure their perimeter, area, and volume in British and Metric measurements.

Unit Goals and Outcomes-Students will be able to:

* Recognize and draw triangles identified by sides or angles.
* Recognize the properties and attributes of quadrilaterals and composite figures.
* Identify cubes, rectangular prisms, cones and cylinders
* Measure and estimate lengths in standard units
* Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures
* Geometric measurement: understand concepts of area and relate area to multiplication and addition
* Recognize volume as an attribute of solid figures and understand concepts of volume measurement.
* Apply perimeter, area, and volume formulas to tasks and real world problems.
* Use appropriate tools of measurement.

Priority Standards:

Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures. (4.G.1)

Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems. (6.G.1)

Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale. (4.MD.2)

Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.* (4.MD.3)

Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:

a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a “one-degree angle,” and can be used to measure angles.

b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees. (4.MD.5)

Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure. (4.MD.6)

Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure. (4.MD.7)

Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

1. A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume.
2. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units. (5.MD.3)

Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units*.*

(5.MD.4)

Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume*.* (5.MD.5)

* Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication. (5.MD.5a)
* Apply the formulas *V* = *l* × *w* × *h* and *V* = *b* × *h* for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems. (5.MD.5b)
* Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems*.* (5.MD.5c)

Coherence:

Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.* (5.G.3)

Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter- circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.12 (1.G.2)

Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.17 Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. (2.G.1)

Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories. (3.G.1)

12Students do not need to learn formal names such as “right rectangular prism.”

17Sizes are compared directly or visually, not compared by measuring.

Standards for Math Practice:

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MP.3 Construct viable arguments and critique the reasoning of others.

MP.4 Model with mathematics.

MP.5 Use appropriate tools strategically.

MP.6 Attend to precision.

MP.7 Look for and make use of structure.

MP.8 Look for and express regularity in repeated reasoning.

Assessment of Priority Standards and Key Content:

|  |  |
| --- | --- |
| Recognize and draw triangles identified by sides or angles. |  |
| Recognize the properties and attributes of quadrilaterals and composite figures. |  |
| Identify cubes, rectangular prisms, cones and cylinders |  |
| Measure and estimate lengths in standard units |  |
| Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures |  |
| Geometric measurement: understand concepts of area and relate area to multiplication and addition |  |
| Recognize volume as an attribute of solid figures and understand concepts of volume measurement. |  |
| Apply perimeter, area, and volume formulas to tasks and real world problems. |  |
| Use appropriate tools of measurement. |  |

Main Texts and Tools of Instruction:

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| --- | --- |
| The site below demonstrates measuring with a meter stick.  [*http://classroom.ldisd.net/users/0335/docs/how\_to\_use\_a\_meter\_stick.pdf*](http://classroom.ldisd.net/users/0335/docs/how_to_use_a_meter_stick.pdf) |  |

<https://www.illustrativemathematics.org/>

<http://www.math-aids.com/>

<http://www.commoncoresheets.com/Values.php>

<http://www.mhschool.com/math/mathconnects/wa/assets/docs/398_401_wa_gr4_adllsn_onln.pdf>

Pages 200 -227, Contemporary’s NUMBER POWER REVIEW, Whole Numbers to Algebra, Robert Mitchell, 1993

ABE MATHEMATICS SCOPE AND SEQUENCE

**Level Two-Unit Seven** GLE 4 - 6

Teachers: Maura McCabe, Ryan Lamey, and Stacy Decker

Unit Title: Statistics and Probability

Amount of time for the unit: 12 hours

Purpose of Unit: The purpose of this unit is to provide activities for data collection and summary through line plots, tables, and graphs. Although the CCRSAE does not provide explicit standards for line, circle or pictographs, we include them at this level and distinguish between the appropriate uses of each. Mean, median, mode and range are taught as measures of central tendencies and spread.

Unit Goals and Outcomes-Students will be able to:

* Conduct a survey on a statistical question and display the results (data sets) on line plots and histograms
* Calculate the mean, median, mode, and range for a set of data.
* Measure lengths to the nearest quarter inch and record the results on a line plot.
* Read and interpret bar, line, circle and pictographs.
* Make the appropriate selection of bar, line or circle graph to represent and interpret data.

Priority Standards:

Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, “How old am I?” is not a statistical question, but “How old are the students in my school?” is a statistical question because one anticipates variability in students’ ages.* (6.SP.1)

Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape. (6.SP.2)

Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number. (6.SP.3)

Display numerical data in plots on a number line, including dot plots, histograms, and box plots. (6.SP.4) [Also see S.ID.1]

Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.* (5.MD.2)

[Note from panel: Plots of numbers other than measurements also should be encouraged.]

Coherence:

Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another. (1.MD.4)

Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph. (2.MD.10)

Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step “how many more” and “how many less” problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.* (3.MD.3)

Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters. (3.MD.4)

Standards for Math Practice:

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MP.3 Construct viable arguments and critique the reasoning of others.

MP.4 Model with mathematics.

MP.5 Use appropriate tools strategically.

MP.6 Attend to precision.

MP.7 Look for and make use of structure.

MP.8 Look for and express regularity in repeated reasoning.

Assessment of Priority Standards and Key Content:

**NOTE:** Some of the activities below are repeated to assess more than one goal.

|  |  |
| --- | --- |
| Conduct a survey on a statistical question and display the results (data sets) on line plots and histograms. |  |
| Calculate the mean, median, mode, and range for a set of data. |  |
| Measure lengths to the nearest quarter inch and record the results on a line plot. |  |
| Read and interpret bar, line, circle and pictographs. |  |
| Make the appropriate selection of bar, line or circle graph to represent and interpret data. | Create a graph to record change over time for temperature or precipitation    Online assessment  <http://worksheets.tutorvista.com/choosing-the-appropriate-graph-worksheet.html> |

Main Texts and Tools of Instruction:

|  |  |
| --- | --- |
| *Massachusetts Community College Workforce*  *Development Transformation Agenda* has developed contextualized modules for three key **Industry Sectors**—healthcare, advanced manufacturing and information technology.  <http://mccwdta.etlo.org> |  |
| The *Math Assessment Project* provides lesson plans with formative assessments  <http://www.map.mathshell.org/download.php?fileid=1619> |  |

<https://www.illustrativemathematics.org/>

<http://www.math-aids.com/>

<http://www.commoncoresheets.com/Values.php>

<http://go.hrw.com/resources/go_sc/hst/HSTSW251.PDF>

www.mathworksheetsland.com

Pages 154 – 171, Contemporary’s NUMBER POWER REVIEW, Whole Numbers to Algebra, Robert Mitchell, 1993